

Discrete Mathematics Mid Term Examination

7th September 2026

Time Allowed: 3 Hours

Total Marks: 50

Please attempt all 5 questions. Each question carries 10 marks.

Question 1:

- (a) Prove that for an undirected graph (4)

$$\sum_{n \in V(G)} \deg(v) = 2|E(G)|$$

- (b) Prove that a tree with n vertices has exactly $n - 1$ edges. (3)
- (c) Prove that every tree with at least 2 vertices contains at least two leaves (vertices of degree 1). (3)

Question 2:

- (a) Define an **Eulerian circuit**. Prove that a connected graph contains an Eulerian circuit if and only if every vertex has an even degree. (6)
- (b) State **Dirac's Theorem** regarding Hamiltonian graphs. Give an example of a simple graph that is Hamiltonian but does not satisfy Dirac's condition. (4)

Question 3:

- (a) State **Euler's Formula** for connected planar graphs. Use it to prove that for any simple, connected planar graph with $V \geq 3$ vertices and E edges, $E \leq 3V - 6$. (6)
- (b) Using the inequality derived in part (a), show that the complete graph K_5 is non-planar. (4)

Question 4:

- (a) Prove that every simple planar graph has at least one vertex of degree 5 or less. (3)
- (b) Using the result from part (a), prove the **Six Colour Theorem** for planar graphs - namely that any planar graph is six colourable (7)

Question 5:

- (a) State the **Deletion-Contraction formula** for chromatic polynomials. Use it to find the chromatic polynomial of the cycle graph C_4 . (6)
- (b) State **Brooks' Theorem**. What is the chromatic number of the complete bipartite graph $K_{3,3}$? Briefly justify your answer. (4)